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D. J. Farrell
Department of Mathematics, The University of Auckland, Auckland 1142, New Zealand

W. B. Moors
Department of Mathematics, The University of Auckland, Auckland 1142, New Zealand
w.moors@auckland.ac.nz

An Application of the Generalised James' Weak Compactness Theorem

We provide a short proof of following theorem, due to Delbaen and Orihuela and independently, Pérez-Aros and Thibault. Let A be a nonempty closed and bounded convex subset of a Banach space $(X, \|\cdot\|)$ and let W be a nonempty weakly compact subset of $(X, \|\cdot\|)$. If we have

$$x_0^* \in \{x^* \in X^* : \sup_{a\in A} x^*(a) < 0\} \quad \text{and} \quad \operatorname{argmax}(y^*|_A) \neq \varnothing$$

for each $y^* \in \{x^* \in X^* : \sup_{a\in A} x^*(a) < 0 \text{ and } \sup_{w\in W} |(x^* - x_0^*)(w)| < 1\}$, then A is weakly compact.

Keywords: Weak compactness, James' weak compactness theorem.

MSC: 46B20; 46B26, 49A50, 49A51.