M. Ôtani

Department of Applied Physics, School of Science and Engineering, Waseda University, Tokyo, Japan
`otani@waseda.jp`

V. Staicu

CIDMA – Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro, Aveiro, Portugal
`vasile@ua.pt`

On Some Nonlinear Parabolic Equations with Nonmonotone Multivalued Terms

We study the local and the global existence of solutions to a class of nonlinear parabolic initial-boundary value problems driven by the equation

$$\frac{\partial u\left(x,t\right)}{\partial t} - \triangle u\left(x,t\right) \in -\partial\Phi\left(u\left(x,t\right)\right) + G\left(x,t,u\left(x,t\right)\right), \left(x,t\right) \in Q_T,$$

where $\partial\Phi$ denotes the subdifferential (in the sense of convex analysis) of a proper, convex and lower semicontinuous function $\Phi\colon \mathbb{R} \to [0,\infty]$, $\Omega \subseteq \mathbb{R}^N$ is a bounded open set, $T > 0$, $Q_T := \Omega \times [0,T]$, and $G\colon Q_T \times \mathbb{R} \to 2^{\mathbb{R}}$ is a multivalued mapping whose growth order with respect to u is Sobolev sub-critical.

We prove two local existence results: one for the case where the multivalued mapping $u \mapsto G\left(\cdot,\cdot,u\right)$ is upper semicontinuous with closed convex values and the second one deals with the case when $u \mapsto G\left(\cdot,\cdot,u\right)$ is lower semicontinuous with closed (not necessarily convex) values. We also give two types of results concerning the global continuation of local solutions. One is for any large data and the other one for small data. Finally, we exemplify the applicability of our results.

Keywords: Nonlinear parabolic equations, nonmonotone multivalued terms, initial-boundary value problems, existence results, local solutions, small and large global solutions.

MSC: 35K55, 35K61, 35B60, 35G25.