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**On the Linear Structures Induced by the Four Order Isomorphisms
Acting on $\text{Cvx}_0(\mathbb{R}^n)$**

It is known that the volume functional $\phi \mapsto \int e^{-\phi}$ satisfies certain concavity or convexity inequalities with respect to three of the four linear structures induced by the order isomorphisms acting on $\text{Cvx}_0(\mathbb{R}^n)$. In this note we define the fourth linear structure on $\text{Cvx}_0(\mathbb{R}^n)$ as the pullback of the standard linear structure under the \mathcal{J} transform. We show that, interpolating with respect to this linear structure, no concavity or convexity inequalities hold, and prove that a quasi-convexity inequality is violated only by up to a factor of 2. We also establish all the order relations which the four different interpolations satisfy.

Keywords: Convexity, interpolation, order isomorphisms, duality, Legendre transform, A-transform, J-transform.

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