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Finite-Time Stabilization of Continuous Inertial Dynamics Combining Dry Friction with Hessian-Driven Damping

In a Hilbert space $\mathcal{H}$, we study the stabilization in finite-time of the trajectories generated by a continuous (in time t) damped inertial dynamic system. The potential function $f\colon \mathcal{H} \to \mathbb{R}$ to be minimized is supposed to be differentiable, not necessarily convex. It enters the dynamic via its gradient. The damping results from the joint action of dry friction, viscous friction, and a geometric damping driven by the Hessian of f. The dry friction damping function $\phi\colon \mathcal{H} \to \mathbb{R}_+$, which is convex and continuous with a sharp minimum at the origin (typically $\phi(x) = r\|x\|$ with $r > 0$), enters the dynamic via its subdifferential. It acts as a soft threshold operator on the velocities, and is at the origin of the stabilization property in finite-time. The Hessian driven damping, which enters the dynamics in the form $\nabla^2 f(x(t))\dot{x}(t)$, permits to control and attenuate the oscillations which occur naturally with the inertial effect. We give two different proofs, in a finite dimensional setting, of the existence of strong solutions of this second-order differential inclusion. One is based on a fixed point argument and Leray-Schauder theorem, the other one uses the Yosida approximation technique together with the Mosco convergence. We also give an existence and uniqueness result in a general Hilbert framework by assuming that the Hessian of the function f is Lipschitz continuous on the bounded sets of $\mathcal{H}$. Then, we study the convergence properties of the trajectories as $t \to +\infty$, and show their stabilization property in finite-time. The convergence results tolerate the presence of perturbations (or errors) under the sole assumption of their asymptotic convergence to zero. The study is extended to the case of a nonsmooth convex function f by using Moreau's envelope.

Keywords: Damped inertial dynamics, differential inclusion, dry friction, Hessian-driven damping, finite-time stabilization.

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