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Non-Occurrence of a Gap Between Bounded and Sobolev Functions for a Class of Nonconvex Lagrangians

We consider the classical functional of the Calculus of Variations of the form

$$I(u) = \int_{\Omega} F(x, u(x), \nabla u(x)) dx$$

where Ω is a bounded open subset of \mathbb{R}^n and $F: \Omega \times \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a given Carathéodory function; the admissible functions u coincide with a given Lipschitz function on $\partial\Omega$. We formulate some conditions under which a given function in $\phi + W_0^{1,p}(\Omega)$ with $I(u) < +\infty$ can be approximated by a sequence of functions $u_k \in \phi + W_0^{1,p}(\Omega) \cap L^\infty$ converging to u in the norm of $W^{1,p}$, and such that $I(u_k) \rightarrow I(u)$. The problem is strictly related with the non occurrence of the Lavrentiev gap.

Keywords: Lavrentiev, Lavrentieff, approximation, bounded functions, regularity.

MSC: 49N99; 49N60.