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M. Pozzetta
Dip. di Matematica, Università di Pisa, 56127 Pisa, Italy
pozzetta@mail.dm.unipi.it

A Varifold Perspective on the p-Elastic Energy of Planar Sets

Under suitable regularity assumptions, the p-elastic energy of a planar set $E \subset \mathbb{R}^2$ is defined as

$$\mathcal{F}_p(E) = \int_{\partial E} 1 + |k_{\partial E}|^p \; d\mathcal{H}^1,$$

where $k_{\partial E}$ is the curvature of the boundary ∂E. In this work we use a varifold approach to investigate this energy, that can be well defined on varifolds with curvature. First we show new tools for the study of 1-dimensional curvature varifolds, such as existence and uniform bounds on the density of varifolds with finite elastic energy. Then we characterize a new notion of L^1-relaxation of this energy by extending the definition of regular sets by an intrinsic varifold perspective, also comparing this relaxation with the classical one of G. Bellettini and L. Mugnai [*Characterization and representation of the lower semicontinuous envelope of the elastica functional*, Annales de l'Institut Henri Poincaré (C), Non Linear Analysis 21(6) (2004) 839–880; *A varifolds representation of the relaxed elastica functional*, J. Convex Analysis 14(3) (2007) 543–564]. Finally we discuss an application to the inpainting problem, examples and qualitative properties of sets with finite relaxed energy.

Keywords: Curvature varifolds, p-elastic energy, relaxation.

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