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Higher Order Problems in the Calculus of Variations: Du Bois-Reymond Condition and Regularity of Minimizers

This paper concerns an N-order problem in the calculus of variations of minimizing the functional $\int_a^b \Lambda(t, x(t), \dots, x^{(N)}(t))\mathrm{d}t$, in which the Lagrangian Λ is a Borel measurable, non autonomous, and possibly extended valued function. Imposing some additional assumptions on the Lagrangian, such as an integrable boundedness of the partial proximal subgradients (up to the $(N-2)$-order variable), a growth condition (more general than superlinearity w.r.t. the last variable) and, when the Lagrangian is extended valued, the lower semicontinuity, we prove that the N-th derivative of a reference minimizer is essentially bounded. We also provide necessary optimality conditions in the Euler-Lagrange form and, for the first time for higher order problems, in the Erdmann-Du Bois-Reymond form. The latter can be also expressed in terms of a (generalized) convex subdifferential, and is valid even without requiring neither a particular growth condition nor convexity in any variable.

Keywords: Calculus of variations, minimizer regularity, higher order problems, necessary conditions, Weierstrass inequality, Erdmann-Du Bois-Reymond condition.

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