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Prescribing Tangent Hyperplanes to $C^{1,1}$ and $C^{1,\omega}$ Convex Hypersurfaces in Hilbert and Superreflexive Banach Spaces

Let X denote $\mathbb{R}^n$ or, more generally, a Hilbert space. Given an arbitrary subset C of X and a collection $\mathcal{H}$ of affine hyperplanes of X such that every $H \in \mathcal{H}$ passes through some point $x_H \in C$, and $C = \{x_H : H \in \mathcal{H}\}$, what conditions are necessary and sufficient for the existence of a $C^{1,1}$ convex hypersurface S in X such that H is tangent to S at x_H for every $H \in \mathcal{H}$? In this paper we give an answer to this question. We also provide solutions to similar problems for convex hypersurfaces of class $C^{1,\omega}$ in Hilbert spaces, and for convex hypersurfaces of class $C^{1,\alpha}$ in superreflexive Banach spaces having equivalent norms with moduli of smoothness of power type $1+\alpha$, $\alpha \in (0,1]$.

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