

© 2019 Heldermann Verlag
Journal of Convex Analysis 26 (2019) 1297–1320

M. Fernández-Unzueta

Centro de Investigación en Matemáticas, A.P. 402 Guanajuato, Mexico
maite@cimat.mx

L. F. Higuera-Montaño

Centro de Investigación en Matemáticas, A.P. 402 Guanajuato, Mexico
fher@cimat.mx

Convex Bodies Associated to Tensor Norms

We determine when a convex body in \mathbb{R}^d is the closed unit ball of a reasonable crossnorm on $\mathbb{R}^{d_1} \otimes \cdots \otimes \mathbb{R}^{d_l}$, $d = d_1 \cdots d_l$. We call these convex bodies “tensorial bodies”. We prove that, among them, the only ellipsoids are the closed unit balls of Hilbert tensor products of Euclidean spaces. It is also proved that linear isomorphisms on $\mathbb{R}^{d_1} \otimes \cdots \otimes \mathbb{R}^{d_l}$ preserving decomposable vectors map tensorial bodies into tensorial bodies. This leads us to define a Banach-Mazur type distance between them, and to prove that there exists a Banach-Mazur type compactum of tensorial bodies.

Keywords: Convex body, tensor norm, Minkowski space, Banach-Mazur distance, tensor product of convex sets, linear mappings on tensor spaces.

MSC: 46M05, 52A21, 46N10, 15A69