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Stability Result for the Extremal Grünbaum Distance Between Convex Bodies

In 1963 Grünbaum introduced the following variation of the Banach-Mazur distance for arbitrary convex bodies $K, L \subset \mathbb{R}^n$:

$$d_G(K, L) = \inf\{|r| \ : \ K' \subset L' \subset rK'\}$$

with the infimum taken over all non-degenerate affine images K' and L' of K and L respectively. In 2004 Gordon, Litvak, Meyer and Pajor proved that the maximal possible distance is equal to n, confirming the conjecture of Grünbaum. In 2011 Jiménez and Naszódi asked if the equality $d_G(K, L) = n$ implies that K or L is a simplex and they proved it under the additional assumption that one of the bodies is smooth or strictly convex. The aim of the paper is to give a stability result for a smooth case of the theorem of Jiménez and Naszódi. We prove that for each smooth convex body L there exists $\varepsilon_0(L) > 0$ such that if $d_G(K, L) \geq (1 - \varepsilon)n$ for some $0 \leq \varepsilon \leq \varepsilon_0(L)$, then $d(K, S_n) \leq 1 + 40n^3 r(\varepsilon)$, where S_n is the simplex in $\mathbb{R}^n$, $r(\varepsilon)$ is a specific function of ε depending on the modulus of the convexity of the polar body of L and d is the usual Banach-Mazur distance. As a consequence, we obtain that for arbitrary convex bodies $K, L \subset \mathbb{R}^n$ their Banach-Mazur distance is less than $n^2 - 2^{-22}n^{-7}$.

Keywords: Banach-Mazur distance, Grünbaum distance, convex body, stability, John's decomposition.

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