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On Convexity and ψ-Uniform Convexity of G-Invariant Functions on an Eaton Triple

An Eaton triple is an algebraic system related to a decomposition statement for vectors of an inner product space V and to some special inner product inequality connected with this decomposition. The Spectral Decomposition for the space of Hermitian matrices associated with Fan-Theobald's trace inequality is a typical example of such a situation. In this paper, for a given Eaton triple (V, G, D) and for a function $F\colon V \to \mathbb{R}$, invariant with respect to the group G acting on V, we study the problem of extending convexity of F from the convex cone $D \subset V$ to the space V. In our approach we reduce the problem from E-system (V, G, D) to its subsystem (W, H, E). Thus we obtain some results related to theorems due to J. von Neumann, C. Davis, A. S. Lewis and T.-Y. Tam et al. Analogous problems are discussed for ψ-uniform convex functions and c-strongly convex functions. Finally, applications are given for matrix spaces endowed with the structure of Eaton triple.

Keywords: Convex function, eigenvalues, singular value, G-invariant function, G-majorization, Eaton triple, normal decomposition system, normal map, psi-uniformly convex function.

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