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On the ℍ-Cone-Functions for H-Convex Sets

Given a compact and H-convex subset K of the Heisenberg group $\mathbb{H}$, with the origin e in its interior, we are interested in finding a homogeneous H-convex function f such that $f(e) = 0$ and $f\big|_{\partial K} = 1$; we will call this function f the ℍ-cone-function of vertex e and base ∂K. While the equivalent version of this problem in the Euclidean framework has an easy solution, in our context this investigation turns out to be quite entangled, and the problem can be unsolvable. The approach we follow makes use of an extension of the notion of convex family introduced by Fenchel. We provide the precise, even if awkward, condition required to K so that ∂K is the base of an ℍ-cone-function of vertex e. Via a suitable employment of this condition, we prove two interesting binding constraints on the shape of the set K, together with several examples.

Keywords: Heisenberg group, H-convexity, convex families, cone-functions.

MSC: 26B25; 53C17, 22E30, 22E25