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"On the Figure of Columns" of Lagrange Revisited

In the design of clamped-clamped circular columns that can accommodate the largest vertical load before buckling, the moment of inertia of the horizontal section is assumed to be proportional to the p-th power of the cross-sectional area for some $p > 0$: $p = 2$ (solid column) and $p = 1$ (hollow column). Existence of maximizing profiles has been established by Cox and Overton [SIAM J. Math. Anal. 23 (1992) 287–325] under the assumption of strictly positive lower and upper bounds on the profiles. Yet, their numerical computations indicate that the bounds are not necessary to get maximizing profiles.

In this paper, we revisit some aspects of the *unconstrained problem* with emphasis on the cases $0 < p \leq 1$. The existence of a maximizing profile in $L^1(0,1)$ is established for $0 < p < 1$ without bounds. The existence theorem of Cox and Overton is extended to continuous profiles in $C[0,1]$. In addition, their strictly positive lower bound assumption is relaxed for $p \leq 1$ in both $L^\infty(0,1)$ and $C[0,1]$. The dual inf sup problem is shown to be bounded above by 48 for $p \leq 1$, but is $+\infty$ for $p > 1$. Additional results are given for *degenerate profiles*, that is, profiles whose lower bound is zero at some points.

Keywords: Lagrange problem, column, buckling, eigenvalue, degenerate differential operator.

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