Journal of Convex Analysis 26 (2019) 739–751

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Weak Compactness of Sublevel Sets in Complete Locally Convex Spaces

We prove that if X is a complete locally convex space and $f\colon X \to \mathbb{R} \cup \{+\infty\}$ is a function such that $f - x^*$ attains its minimum for every $x^* \in U$, where U is an open set with respect to the Mackey topology in X^*, then for every $\gamma \in \mathbb{R}$ and $x^* \in U$ the set $\{x \in X : f(x) - \langle x^*, x\rangle \leq \gamma\}$ is relatively weakly compact. This result corresponds to an extension of Theorem 2.4 in a recent paper of J. Saint Raymond [Mediterr. J. Math. 10(2) (2013) 927–940]. Directional James compactness theorems are also derived.

Keywords: Convex functions, conjugate functions, inf-convolution, epi-pointed functions, weak compactness, inf-compact functions.

MSC: 46A25, 46A04, 46A50