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On a Decomposition Formula for the Proximal Operator of the Sum of Two Convex Functions

The main result of the present theoretical paper is an original decomposition formula for the proximal operator of the sum of two proper, lower semicontinuous and convex functions f and g. For this purpose, we introduce a new operator, called *f-proximal operator of g* and denoted by prox_g^f, that generalizes the classical notion. Then we prove the decomposition formula $\mathrm{prox}_{f+g} = \mathrm{prox}_f \circ \mathrm{prox}_g^f$. After collecting several properties and characterizations of prox_g^f, we prove that it coincides with the fixed points of a generalized version of the classical Douglas-Rachford operator. This relationship is used for the construction of a weakly convergent algorithm that computes numerically this new operator prox_g^f, and thus, from the decomposition formula, allows to compute numerically prox_{f+g}. It turns out that this algorithm was already considered and implemented in previous works, showing that prox_g^f is already present (in a hidden form) and useful for numerical purposes in the existing literature. However, to the best of our knowledge, it has never been explicitly expressed in a closed formula and neither been deeply studied from a theoretical point of view. The present paper contributes to fill this gap in the literature. Finally we give an illustration of the usefulness of the decomposition formula in the context of sensitivity analysis of linear variational inequalities of second kind in a Hilbert space.

Keywords: Convex analysis, proximal operator, Douglas-Rachford operator, forward-backward operator.

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