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S. Drapeau
Shanghai Jiao Tong University, School of Mathematical Sciences, and: China Academy of Financial Research, 211 West Huaihai Road, Shanghai, China
sdrapeau@saif.sjtu.edu.cn

A. Jamneshan
Dept. of Mathematics and Statistics, University of Konstanz, 78464 Konstanz, Germany
asgar.jamneshan@uni-konstanz.de

M. Kupper
Dept. of Mathematics and Statistics, University of Konstanz, 78464 Konstanz, Germany
kupper@uni-konstanz.de

A Fenchel-Moreau Theorem for $\bar{L}^0$-Valued Functions

We establish a Fenchel-Moreau type theorem for proper convex functions $f\colon X \to \bar{L}^0$, where $(X, Y, \langle\cdot,\cdot\rangle)$ is a dual pair of Banach spaces and $\bar{L}^0$ is the space of all extended real-valued functions on a σ-finite measure space. We introduce the concept of stable lower semi-continuity which is shown to be equivalent to the existence of a dual representation

$$f(x) = \sup_{y \in L^0(Y)} \{\langle x, y\rangle - f^*(y)\}, \quad x \in X,$$

where $L^0(Y)$ is the space of all strongly measurable functions with values in Y, and $\langle\cdot,\cdot\rangle$ is understood pointwise almost everywhere. The proof is based on a conditional extension result and conditional functional analysis.

Keywords: Fenchel-Moreau theorem, vector duality, semi-continuous extension, conditional functional analysis.

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