© 2019 Heldermann Verlag Journal of Convex Analysis 26 (2019) 515–526

E. Jordá

Dep. de Matemática Aplicada, Universidad Politécnica de Valencia, Plaza Ferrándiz y Carbonell 2, 03801 Alcoy, Spain ejorda@mat.upv.es

A. M. Zarco

Dep. de Matemática Aplicada, Universidad Politécnica de Valencia, Plaza Ferrándiz y Carbonell 2, 03801 Alcoy, Spain anzargar@upv.es

Smoothness in some Banach Spaces of Operators and Vector Valued Functions

A well known criterion of Šmulyan states that the norm $\|\cdot\|$ of a real Banach space X is Gâteaux differentiable at $x \in X$ if and only if there is $x^* \in S_{X^*}$ which is w^* -exposed by x in B_{X^*} and that the norm is Fréchet differentiable at x if and only if there is $x^* \in S_{X^*}$ which is w^* -strongly exposed in B_{X^*} by x. We show that in this criterion B_{X^*} can be replaced by a convenient smaller set, and we apply this extended criterion to characterize the points of Gâteaux and Fréchet differentiability of the norm in epsilon products of Banach spaces, extending previous work of Heinrich. As a consequence we get some results of smoothness of the norm in some Banach spaces of continuous and harmonic vector valued functions.

Keywords: Banach spaces, Frechet and Gateaux differentiability, epsilon products.

MSC: 46B20, 46B50