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Midsets and Voronoi Type Decomposition with Respect to Closed Convex Sets

Let Ω_k denote the collection of all nonempty closed convex subsets of $\mathbb{R}^k$. We provide short proofs for the following: (i) $\{x \in \mathbb{R}^k : dist(x, A) = \varepsilon\}$ is a C^1-manifold of dimension $k-1$ for every $A \in \Omega_k \setminus \{\mathbb{R}^k\}$ and $\varepsilon > 0$, (ii) $\{x \in \mathbb{R}^k : dist(x, A) = dist(x, B)\}$ is a C^1-manifold of dimension $k-1$ for any two disjoint $A, B \in \Omega_k$. We also study the distance of points in $\mathbb{R}^k$ to finitely many closed convex sets. Let $k, n \geq 2$ and $A = \bigcup_{j=1}^n A_j$, where $A_1, \ldots, A_n \in \Omega_k$ are pairwise disjoint. We consider a Voronoi type decomposition of $\mathbb{R}^k$ and establish some topological properties of its 'conflict set'. Letting $X_p = \{x \in \mathbb{R}^k : |\{a \in A : \|x - a\| = dist(x, A)\}| = p\}$, we prove with the help of result (ii) stated above that $X_1 \cup X_2$ is a connected dense open subset of $\mathbb{R}^k$ and that $\overline{X_2} = \bigcup_{p=2}^n X_p$.

Keywords: Euclidean geometry, closed convex sets, Voronoi decomposition, midsets.

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