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Conic James' Compactness Theorem

The following results is proved:
Let A be a convex bounded non weakly relatively compact subset of a Banach space E. We consider a convex weakly compact subset D of E which does not contain the origin.
Then there is a sequence $\{x_n^*\}_{n\geq 1}$ in B_{E^*} and $g_0^* \in \mathrm{co}_\sigma\{x_n^* : n \geq 1\}$ such that for all $h \in \ell_\infty(A)$ satisfying that for all $a \in A$,

$$\liminf_{n\geq 1} x_n^*(a) \leq h(a) \leq \limsup_{n\geq 1} x_n^*(a),$$

we have that $g_0^* - h$ does not attain its supremum on A and $(g_0^* - h)(d) > 0$ for every $d \in D$.

Keywords: James' compactness theorem, weakly compact set.

MSC: 46A50, 46B50