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A New Class of Sets Regularity

Let $A \subset \mathbb{R}^n$ be a closed set and let $S \subset \mathbb{R}^n$ be a set containing A. In this paper we study a new regularity class for A, called *S*-convexity, introduced by C. Nour, H. Saoud and J. Takche [*Regularization via sets satisfying the interior sphere* condition, J. Convex Analysis 25(1) (2018)], where an inner approximation of a closed set by sets satisfying the interior sphere condition is given. We prove that this new class covers several known regularity properties including the proximal smoothness, the exterior sphere condition and the union of closed balls. As an application of such results, we provide a new sufficient condition for the equivalence between proximal smoothness and the exterior sphere condition studied previously by C. Nour, R. J. Stern and J. Takche [*Proximal smoothness and the exterior sphere condition*, J. Convex Analysis 16(2) (2009) 501–514].

Keywords: S-convexity, proximal smoothness, exterior sphere condition, union of closed balls property, proximal analysis, nonsmooth analysis.

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