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Directional Convexity and Characterizations of Beta and Gamma Functions

The logarithmic convexity of restrictions of the Beta function to rays parallel to the main diagonal and the functional equation

$$\varphi(x+1) = \frac{x(x+k)}{(2x+k+1)(2x+k)}\,\phi(x), \quad x > 0,$$

for $k > 0$ allow to get a characterization of the Beta function. This fact and the notion of the beta-type function lead to a new characterization of the Gamma function.

Keywords: Gamma function, Beta function, beta-type function, logarithmical convexity, geometrical convexity, directional convexity, functional equation.

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