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On Semiconcavity via the Second Difference

Let f be a continuous real function on a convex subset of a Banach space. We study what can be said about the semiconcavity (with a general modulus) of f, if we know that the estimate $\Delta_h^2(f,x) \leq \omega(\|h\|)$ holds, where $\Delta_h^2(f,x) = f(x+2h) - 2f(x+h) + f(x)$ and $\omega : [0,\infty) \to [0,\infty)$ is a nondecreasing function right continuous at 0 with $\omega(0) = 0$. A partial answer to this question was given by P. Cannarsa and C. Sinestrari (2004); we prove versions of their result, which are in a sense best possible. We essentially use methods of A. Marchaud, S. B. Stechkin and others, whose results clarify when the inequality $|\Delta_h^2(f,x)| \leq \omega(\|h\|)$ implies that f is a C^1 function (and f' is uniformly continuous with a corresponding modulus of continuity).

Keywords: Semiconcave function with general modulus, second difference, second modulus of continuity, Jensen semiconcave function, alpha-midconvex function, semi-Zygmund class.

MSC: 26B25; 46T99