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On Countable Tightness and the Lindelöf Property in Non-Archimedean Banach Spaces

Let $\mathbb{K}$ be a non-archimedean valued field and let E be a non-archimedean Banach space over $\mathbb{K}$. By E_w we denote the space E equipped with its weak topology and by $E^*_{w^*}$ the dual space E^* equipped with its weak* topology. Several results about countable tightness and the Lindelöf property for E_w and $E^*_{w^*}$ are provided. A key point is to prove that for a large class of infinite-dimensional polar Banach spaces E, countable tightness of E_w or $E^*_{w^*}$ implies separability of $\mathbb{K}$. As a consequence we obtain the following two characterizations of the field $\mathbb{K}$:

(a) A non-archimedean valued field $\mathbb{K}$ is locally compact if and only if for every Banach space E over $\mathbb{K}$ the space E_w has countable tightness if and only if for every Banach space E over $\mathbb{K}$ the space $E^*_{w^*}$ has the Lindelöf property.

(b) A non-archimedean valued separable field $\mathbb{K}$ is spherically complete if and only if every Banach space E over $\mathbb{K}$ for which E_w has the Lindelöf property must be separable if and only if every Banach space E over $\mathbb{K}$ for which $E^*_{w^*}$ has countable tightness must be separable.

Both results show how essentially different are non-archimedean counterparts from the "classical" corresponding theorems for Banach spaces over the real or complex field.

Keywords: Non-archimedean Banach spaces, weak topology, Lindelöf property, countable tightness.

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