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Clarke and Limiting Subdifferentials of Integral Functionals

Some comparisons (equality, strong density or complete characterizations) are made between the Clarke and limiting subdifferentials for the class of integral functionals defined on reflexive Lebesgue spaces L_p, without any locally Lipschitzian assumptions on these functionals. Let I_f be the integral functional associated with a normal integrand $f : \Omega \times E \to \mathbb{R} \cup \{\pm\infty\}$, where E is a finite dimensional space and Ω is endowed with an atomless σ-finite measure. At a given point x such that almost everywhere the Clarke subdifferential of f_ω at $x(\omega)$ coincide with the closed convex hull of the limiting subdifferential and such that the limiting subdifferential multifunction at x, $\omega \rightrightarrows \partial^L f_\omega(x(\omega))$, admits a $q = p(p-1)^{-1}$ integrable selection, we show that the limiting subdifferential of the integral functional is strongly dense in the Clarke subdifferential. This property allows us to give an exact computation of the Clarke subdifferential by a legitimate disintegration formula. As a consequence, when for almost every ω in Ω the function $f_\omega(.)$ is locally Lipschitz at $x(\omega)$ then the limiting subdifferential of I_f at x is strongly dense in the Clarke subdifferential of I_f at x. When the Lipschitz rate is q-integrable, there is the coincidence of the Clarke subdifferential with the limiting subdifferential of I_f at x and the disintegration formula is valid. Moreover such a disintegration formula is valid for the calculus of the Clarke normal (respectively tangent) cone at a given point to the set of p-integrable selections of a measurable multifunction.

Keywords: Nonsmooth Analysis, Frechet, Clarke, limiting subdifferentials, non locally Lipschitzian integral functionals.

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