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Positively α-Far Sets and Existence Results for Generalized Perturbed Sweeping Processes

We consider the general class of positively α-far sets, introduced by T. Haddad, A. Jourani and L. Thibault [*Reduction of sweeping process to unconstrained differential inclusion*, Pac. J. Optim. 4 (2008) 493–512], which contains strictly the class of uniformly prox-regular sets and the class of uniformly subsmooth sets. We provide some conditions to assure the uniform subsmoothness, and thus the positive α-farness, of the inverse images under a differentiable mapping. Then, we take advantage of the properties of this class to study the generalized perturbed sweeping process

$$\begin{cases} -\dot{x}(t) \in F(t,x(t)) + g(x(t))N\left(C(t),h(x(t))\right) & \text{a.e. } t \in [T_0,T], \\ x(T_0) = x_0 \in h^{-1}\left(C(T_0)\right), \end{cases}$$

where $g\colon X \to \mathcal{L}(Y;X)$, $h\colon X \to Y$ are two functions, X, Y are two separable Hilbert spaces and the sets $C(t)$ belong to the class of positively α-far sets. This differential inclusion includes the classical perturbed sweeping process as well as complementarity dynamical systems. Our study is achieved by approximating the given differential inclusion with maximally perturbed differential inclusions which, under certain compactness conditions, converges to an absolutely continuous solution. Moreover, this approach allows us to get existence for evolution inclusions of the form

$$\begin{cases} -\dot{x}(t) \in \partial f(t,x(t)) + F(t,x(t)) & \text{a.e. } t \in [T_0,T], \\ x(T_0) = x_0, \end{cases}$$

where $[T_0, T]$ is a fixed interval with $0 \leq T_0 < T$, $f\colon [T_0, T] \times X \to \mathbb{R} \cup \{+\infty\}$ is a lower semicontinuous function, not necessarily convex. Here $\partial f(t, \cdot)$ denotes the Clarke subdifferential of the function $f(t, \cdot)$ and $F\colon [T_0, T] \times X \rightrightarrows X$ is a perturbation term.