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Characterizing P-spaces X in Terms of $C_p(X)$

Dual weak barrelledness led us to prove that X is a P-space if and only if every pointwise eventually zero sequence in $C_p(X)$ is summable, and other better known characterizations. Novel ones recall utility functions from economics and Arkhangel'skii's (strict) τ-continuity. Mackey $\aleph_0$-barrelled duality leads us to prove that X is discrete if and only if every bounded σ-compact set in $C_p(X)$ is relatively compact. We relax the σ-compact hypothesis of Velichko and the σ-countably compact hypothesis of Tkachuk/Shakhmatov to prove: *X is a P-space if and only if $C_p(X)$ is σ-relatively sequentially complete.*

Keywords: P-spaces, relatively compact, weak barrelledness.

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