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# Ball Proximinal and Strongly Ball Proximinal Spaces

Let $Y$ be an $E$-proximinal (respectively, a strongly proximinal) subspace of $X$. We prove that $Y$ is (strongly) ball proximinal in $X$ if and only if for any $x \in X$ with $(x+Y) \cap B_X \neq \emptyset$, $(x+Y) \cap B_X$ is (strongly) proximinal in $x+Y$. Using this characterization and a smart construction, we obtain three Banach spaces $Z \subset Y \subset X$ such that $Z$ is ball proximinal in $X$ and $Y/Z$ is ball proximinal in $X/Z$, but $Y$ is not ball proximinal in $X$. This solves a problem raised by P. Bandyopadhyay, Bor-Luh Lin and T.S.S.R.K. Rao [*Ball proximinality in Banach spaces,* in: Banach Spaces and Their Applications in Analysis (Oxford/USA, 2006) B. Randrianantoanina et al (eds.) Proceedings in Mathematics, de Gruyter, Berlin (2007) 251–264].

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