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Properties of Hadamard Directional Derivatives: Denjoy-Young-Saks Theorem for Functions on Banach Spaces

The classical Denjoy-Young-Saks theorem on Dini derivatives of arbitrary functions $f : \mathbb{R} \to \mathbb{R}$ was extended by U.S. Haslam-Jones (1932) and A.J. Ward (1935) to arbitrary functions on $\mathbb{R}^2$. This extension gives the strongest relation among upper and lower Hadamard directional derivatives $f_H^+(x, v)$, $f_H^-(x, v)$ ($v \in X$) which holds almost everywhere for an arbitrary function $f : \mathbb{R}^2 \to \mathbb{R}$. Our main result extends the theorem of Haslam-Jones and Ward to functions on separable Banach spaces.

Keywords: Hadamard upper and lower directional derivatives, Denjoy-Young-Saks theorem, separable Banach space, Hadamard differentiability, Frechet differentiability, Hadamard subdifferentiability, Frechet subdifferentiability, Gamma-null set, Aronszajn null set.

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