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# Uniform Convexity of Paranormed Generalizations of $L^p$ Spaces

For a measure space $(\Omega, \Sigma, \mu)$ and a bijective increasing function $\varphi : [0, \infty) \to [0, \infty)$ the $L^p$-like paranormed ($F$-normed) function space with the paranorm of the form $\mathbf{p}_\varphi(x) = \varphi^{-1}\left(\int_\Omega \varphi \circ |x| \, d\mu\right)$ is considered. Main results give general conditions under which this space is uniformly convex. The Clarkson theorem on the uniform convexity of $L^p$-space is generalized. Under some specific assumptions imposed on $\varphi$ we give not only a proof of the uniform convexity but also show the formula of a modulus of convexity. We establish the uniform convexity of all finite-dimensional paranormed spaces, generated by a strictly convex bijection $\varphi$ of $[0, \infty)$. However, the *a contrario* proof of this fact provides no information on a modulus of convexity of these spaces. In some cases it can be done, even an exact formula of a modulus can be proved. We show how to make it in the case when $S = \mathbb{R}^2$ and $\varphi$ is given by $\varphi(t) = \mathrm{e}^t - 1$.

**Keywords**: Lp-like paranorm, paranormed space, uniformly convex paranormed space, modulus of convexity, convex function, geometrically convex function, superquadratic function.

**MSC**: 46A16, 46E30; 47H09, 47H10