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Gâteaux and Hadamard Differentiability via Directional Differentiability

Let X be a separable Banach space, Y a Banach space and $f : X \to Y$ an arbitrary mapping. Then the following implication holds at each point $x \in X$ except a σ-directionally porous set: If the one-sided Hadamard directional derivative $f'_{H+}(x,u)$ exists in all directions u from a set $S_x \subset X$ whose linear span is dense in X, then f is Hadamard differentiable at x. This theorem improves and generalizes a recent result of A. D. Ioffe, in which the linear span of S_x equals X and $Y = \mathbb{R}$. An analogous theorem, in which f is pointwise Lipschitz, and which deals with the usual one-sided derivatives and Gâteaux differentiability is also proved. It generalizes a result of D. Preiss and the author, in which f is supposed to be Lipschitz.

Keywords: Gateaux differentiability, Hadamard differentiability, directional derivatives, Hadamard directional derivatives, sigma-directionally porous set, pointwise Lipschitz mapping.

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