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Approximation by DC Functions and Application to Representation of a Normed Semigroup

Let Ω be a nonempty compact set of a locally convex space L, and let $C(\Omega)$ be the Banach space of all real-valued continuous functions on Ω endowed with the sup-norm. In this paper, we show first that for every $f \in C(\Omega)$, and for every $\varepsilon > 0$, there are continuous affine functions $(g_i)_{i=1}^m, (h_j)_{j=1}^n$ on L for some $m, n \in \mathbb{N}$ such that

$$|f(\omega) - [(g_1 \vee g_2 \vee \cdots \vee g_m) - (h_1 \vee h_2 \vee \cdots \vee h_n)](\omega)| < \varepsilon$$

uniformly for $\omega \in \Omega$. We prove then that if $\Omega = B_{X^*}$, the closed unit ball of X^* of a Banach space X endowed with the w^*-topology, then $C(\Omega)^*$ is just the dual of the normed semigroup $\mathrm{b}(X)$ generated closed balls in X.