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Conic Separation of Finite Sets. I: The homogeneous case

This work addresses the issue of separating two finite sets in $\mathbb{R}^n$ by means of a suitable revolution cone

$$\Gamma(z, y, s) = \{x \in \mathbb{R}^n : s\,\|x - z\| - y^T(x - z) = 0\}.$$

The specific challenge at hand is to determine the aperture coefficient s, the axis y, and the apex z of the cone. These parameters have to be selected in such a way as to meet certain optimal separation criteria. Part I of this work focusses on the homogeneous case in which the apex of the revolution cone is the origin of the space. The homogeneous case deserves a separated treatment, not just because of its intrinsic interest, but also because it helps to built up the general theory. Part II of this work concerns the non-homogeneous case in which the apex of the cone can move in some admissible region. The non-homogeneous case is structurally more involved and leads to challenging nonconvex nonsmooth optimization problems.

Keywords: Conical separation, revolution cone, convex optimization, DC-optimization, proximal point techniques, classification.

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