Journal of Convex Analysis 20 (2013) 1013–1024

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A Generalization of Blaschke's Convergence Theorem in Metric Spaces

A metric space (X, d) together with a set-valued mapping $G : X \times X \to 2^X$ is said to be a *generalized segment space* (X, d, G) if $G(x, y) \neq \emptyset$ for all $x, y \in X$ and for any sequences $x_n \to x$ and $y_n \to y$ in X, $d_H\big(G(x_n, y_n), G(x, y)\big) \to 0$ as $n \to \infty$, where d_H is the Hausdorff distance. Normed linear spaces, nonempty convex sets, and proper uniquely geodesic spaces, etc are generalized segment spaces for suitable G. A subset A of X is called *G-type convex* if $G(x, y) \subset A$ whenever $x, y \in A$. We prove a generalization of Blaschke's convergence theorem for metric spaces: if (X, d, G) is a proper generalized segment space, then every uniformly bounded sequence of nonempty G-type convex subsets of X contains a subsequence which converges to some nonempty compact G-type convex subset in X.

Keywords: Blaschke's convergence theorem, convex sets, generalized convexity, geodesic convex sets, geodesic segments, Hausdorff distance, uniquely geodesic spaces.

MSC: 52A10, 52B55, 52C45