Journal of Convex Analysis 20 (2013) 617–653

S. Masnou
Institut Camille Jordan, Université de Lyon 1, 43 bd du 11 novembre 1918, 69622 Villeurbanne-Cedex, France
masnou@math.univ-lyon1.fr

G. Nardi
Laboratoire Jacques-Louis Lions, Université P. et M. Curie Paris 6, F-75005 Paris, France
nardi@ann.jussieu.fr

A Coarea-Type Formula for the Relaxation of a Generalized Elastica Functional

We consider the *generalized elastica functional* defined on $\mathrm{L}^1(\mathbb{R}^2)$ as

$$F(u) = \begin{cases} \displaystyle\int_{\mathbb{R}^2} |\nabla u|(\alpha + \beta|\operatorname{div} \frac{\nabla u}{|\nabla u|}|^p)\, dx, & \text{if } u \in \mathrm{C}^2(\mathbb{R}^2), \\ +\infty & \text{else,} \end{cases}$$

where $p > 1$, $\alpha > 0$, $\beta \geq 0$. We study the L^1-lower semicontinuous envelope $\overline{F}$ of F and we prove that, for any $u \in \mathrm{BV}(\mathbb{R}^2)$, $\overline{F}(u)$ can be represented by a coarea-type formula involving suitable collections of $\mathrm{W}^{2,p}$ curves that cover the essential boundaries of the level sets $\{x,\, u(x) > t\}$, $t \in \mathbb{R}$.