Journal of Convex Analysis 20 (2013) 573–598

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Smallness of Singular Sets of Semiconvex Functions in Separable Banach Spaces

Let X be a separable superreflexive Banach space and f be a semiconvex function (with a general modulus) on X. For $k \in \mathbb{N}$, let $\Sigma_k(f)$ be the set of points $x \in X$, at which the Clarke subdifferential $\partial f(x)$ is at least k-dimensional. Note that $\Sigma_1(f)$ is the set of all points at which f is not Gâteaux differentiable. Then $\Sigma_k(f)$ can be covered by countably many Lipschitz surfaces of codimension k which are described by functions, which are differences of two semiconvex functions. If X is separable and superreflexive Banach space which admits an equivalent norm with modulus of smoothness of power type 2 (e.g., if X is a Hilbert space or $X = L^p(\mu)$ with $2 \leq p$), we give, for a fixed modulus ω and $k \in \mathbb{N}$, a complete characterization of those $A \subset X$, for which there exists a function f on X which is semiconvex on X with modulus ω and $A \subset \Sigma_k(f)$. Namely, $A \subset X$ has this property if and only if A can be covered by countably many Lipschitz surfaces S_n of codimension k which are described by functions, which are differences of two Lipschitz semiconvex functions with modulus $C_n\omega$.

Keywords: Semiconvex function with general modulus, Clarke subdifferential, singular set, singular point of order k, Lipschitz surface, DSC surface, superreflexive space.

MSC: 49J52; 46G05