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Strongly Midquasiconvex Functions

Let V be a nonempty convex subset of a normed space X and let $\varepsilon > 0$ and $p > 0$ be given. A function $f : V \to \mathbb{R}$ is called (ε, p)-*strongly midquasiconvex* if

$$f(\frac{x+y}{2}) \leq \max[f(x), f(y)] - \varepsilon(\frac{\|x-y\|}{2})^p \quad \text{for} \quad x, y \in V.$$

We call f p-strongly midquasiconvex if it is (ε, p)-strongly midquasiconvex with a certain $\varepsilon > 0$. We show that if either $p < 1$ and $\dim V = 1$ or $p < 2$ and $\dim V > 1$ then there are no p-strongly midquasiconvex functions defined on V. On the other hand if X is an inner product space with $\dim X \geq 2$, $p \geq 2$, then there exists an $(1, p)$-strongly midquasiconvex function defined on an arbitrary ball in X. Consequently, the case when $p = 1$ and $\dim V = 1$ is of a special interest. Under this assumptions we characterize lower semicontinuous 1-strongly midquasiconvex functions.

Keywords: Quasiconvexity, midquasiconvex function, strongly midquasiconvex function.

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