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Convex Radiant Costarshaped Sets and the Least Sublinear Gauge

The paper studies convex radiant sets (i.e. containing the origin) of a linear normed space X and their representation by means of a gauge. By gauge of a convex radiant set $C \subseteq X$ we mean a sublinear function $p : X \to \bar{\mathbb{R}}$ such that $C = [p \leq 1]$. Besides the most important instance, namely the Minkowski gauge $\mu_C(x) = \inf\{\lambda > 0 : x \in \lambda C\}$, the set C may have other gauges, which are necessarily lower than μ_C. We characterize the class of convex radiant sets which admit a gauge different from μ_C in two different way: they are contained in a translate of their recession cone or, equivalently, they are costarshaped, that is complement of a starshaped set. We prove that the family of all sublinear gauges of a convex radiant set admits a least element and characterize its support set in terms of polar sets. The key concept for this study is the outer kernel of C, that is the kernel (in the sense of Starshaped Analysis) of the complement of C. We also devote some attention to the relation between costarshaped and hyperbolic convex sets.

Keywords: Convex sets, Minkowski gauge, sublinear gauge, radiant sets, costarshaped sets, kernel, outer kernel, polar set, reverse polar, hyperbolic convex sets.

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