M. D. Acosta
Dep. de Análisis Matemático, Universidad de Granada, 18071 Granada, Spain
`dacosta@ugr.es`

A. Kamińska
Dept. of Mathematical Sciences, The University of Memphis, Memphis, TN 38152, U.S.A.
`kaminska@memphis.edu`

M. Mastyło
Institute of Mathematics, Polish Academy of Sciences, Umultowska 87, 61-614 Poznan, Poland
`mastylo@amu.edu.pl`

The Daugavet Property and Weak Neighborhoods in Banach Lattices

The Daugavet property and the diameters of relatively weakly open subsets of unit balls in Banach lattices X on measure spaces are studied. It is shown that under mild assumptions the subspace X_a of order continuous elements inherits the Daugavet property from X. This is applied to prove that if X has the Daugavet property and the Köthe dual X' is strictly monotone (resp. order continuous) then X' contains a lattice isometric (resp. isomorphic) copy of $L_1(0,1)$. These results yield that a large class of r.i. spaces including several interpolation sums fail the Daugavet property and also that any r.i. space over a finite atomless measure space with the Daugavet property coincide to either L_1 or L_∞. Applications are shown for Orlicz, Lorentz, Marcinkiewicz spaces as well for Nakano spaces. It is established that in most cases these spaces do not enjoy the Daugavet property. However, it is proved that in a large class of Orlicz or Nakano spaces (variable exponent spaces), in particular those induced by fast growing Orlicz functions, all non-empty relatively weakly open subsets of their unit balls have diameter two.

Keywords: Daugavet property, weak neighborhoods, Banach lattices, Orlicz spaces, Lorentz spaces, Marcinkiewicz spaces, Nakano spaces, variable exponent spaces.

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