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c Horizontal Convexity on Carnot Groups

Given a real-valued function c defined on the cartesian product of a generic Carnot group \mathbf{G} and the first layer V_1 of its Lie algebra, we introduce a notion of c horizontal convex (c H-convex) function on \mathbf{G} as the supremum of a suitable family of affine functions; this family is defined pointwisely, and depends strictly on the horizontal structure of the group. This abstract approach provides c H-convex functions that, under appropriate assumptions on c , are characterized by the nonemptiness of the c H-subdifferential and, above all, are locally H-semiconvex, thereby admitting horizontal derivatives almost everywhere. It is noteworthy that such functions can be recovered via a Rockafellar technique, starting from a suitable notion of c H-cyclic monotonicity for maps. In the particular case where $c(g, v) = \langle \xi_1(g), v \rangle$, we obtain the well-known weakly H-convex functions introduced by Danielli, Garofalo and Nhieu. Finally, we suggest a possible application to optimal mass transportation.

Keywords: Carnot group, horizontal convexity, c horizontal convexity, c horizontal differential, c horizontal cyclic monotonicity.

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