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# A Differential Characterisation of the Minimax Inequality

We prove the following result: let $K \subseteq \mathbb{R}^N$ be convex with nonempty interior, $X$ a topological space and $f\colon K \times X \to \mathbb{R}$ be concave and u.s.c. in the first variable and coercive and l.s.c. in the second. Then the (perturbed) strict minimax inequality

$$\sup_{\lambda \in K} \inf_{x \in X} f(\lambda, x) + g(\lambda) < \inf_{x \in X} \sup_{\lambda \in K} f(\lambda, x) + g(\lambda),$$

for some continuous concave $g\colon K \to \mathbb{R}$, is equivalent to the following condition on superdifferentials: if $F(\lambda) = \inf_X f(\lambda, x)$, for some $\lambda \in \mathring{K}$

$$\partial F(\lambda) \setminus \bigcup_{\substack{x \in X \\ f(\lambda,x)=F(\lambda)}} \partial f(\lambda, x) \neq \emptyset.$$

As an application of this differential characterisation we prove a generalised version of a theorem of Ricceri, a criterion of regularity for marginal functions, and the fact that to check whether some perturbed minimax inequality holds, one can test with affine perturbation only.

**Keywords**: Minimax inequality, concave functions, marginal functions, multiple solutions to variational problems, nonlinear eigenvalues.