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Generic Fréchet Differentiability on Asplund Spaces via A.E. Strict Differentiability on Many Lines

We prove that a locally Lipschitz function on an open subset G of an Asplund space X, whose restrictions to "many lines" are essentially smooth (i.e., almost everywhere strictly differentiable), is generically Fréchet differentiable on X. In this way we obtain new proofs of known Fréchet differentiability properties of approximately convex functions, Lipschitz regular functions, saddle (or biconvex) Lipschitz functions, and essentially smooth functions (in the sense of Borwein and Moors), and also some new differentiability results (e.g., for partially DC functions). We show that classes of functions $\mathcal{S}_e^g(G)$ and $\mathcal{R}_e^g(G)$ (defined via linear essential smoothness) are respectively larger than classes $\mathcal{S}_e(G)$ (of essentially smooth functions) and $\mathcal{R}_e(G)$ studied by Borwein and Moors, and have also nice properties. In particular, we prove that members of $\mathcal{S}_e^g(G)$ are uniquely determined by their Clarke subdifferentials. We also show the inclusion $\mathcal{S}_e(G) \subset \mathcal{R}_e(G)$ for Borwein-Moors classes.

Keywords: Generic Frechet differentiability, essentially smooth functions, separable reduction.

MSC: 46G05; 46T20