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About the Regularity of Average Distance Minimizers in $\mathbb{R}^2$

We focus on the following irrigation problem introduced by G. Buttazzo, E. Oudet and E. Stepanov [Optimal transportation problems with free Dirichlet regions, in: Variational Methods for Discontinuous Structures, Progr. Nonlinear Differential Equations Appl. 51, Birkhäuser, Basel (2002) 41–65]:

$$\min \mathcal{F}(\Sigma) := \int_\Omega dist(x, \Sigma) \, \mathrm{d}\mu(x),$$

where Ω is an open subset of $\mathbb{R}^2$, μ is a probability measure and where the minimum is taken over all the sets $\Sigma \subset \Omega$ such that Σ is compact, connected, and $\mathcal{H}^1(\Sigma) \leq \alpha_0$ for a given positive constant α_0. In this paper we seek for some conditions to find in Σ some pieces of C^1 (or more) regular curves. We prove that it is the case in the ball B when $\Sigma \cap B$ contains no corner points. More generally we prove that the Left and Right tangents half lines of Σ (that exist everywhere out of endpoints and triple points) are semicontinuous. We also discuss how the regularity is linked with the pull back measure $\psi := k\sharp\mu$ where k is the projection on Σ. In particular $\Sigma \cap B$ is $C^{1,\alpha}$ when ψ is regular with respect to $\mathcal{H}^1$ with density in a certain L^p. We also prove that Σ is locally a Lipschitz graph away from triple points and endpoints, and that the mean curvature of Σ is a measure that is explicited in terms of measure ψ.