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Slices in the Unit Ball of the Symmetric Tensor Product of a Banach Space

We prove that every infinite-dimensional C^*-algebra X satisfies that every slice of the unit ball of $\widehat{\bigotimes}_{N,s,\pi} X$ (N-fold projective symmetric tensor product of X) has diameter two. We deduce that every infinite-dimensional Banach space X whose dual is an L_1-space satisfies the same result. As a consequence, if X is either a C^*-algebra or either a predual of an L_1-space, then the space of all N-homogeneous polynomials on X, $\mathcal{P}^N(X)$, is extremely rough, whenever X is infinite-dimensional. If Y is a predual of a von Neumann algebra, then Y is infinite-dimensional if, and only if, every w^*-slice of the unit ball of $\mathcal{P}_I^N(Y)$ (the space of integral N-homogeneous polynomials on Y) has diameter two. As a consequence, under the previous assumptions, the N-fold symmetric injective tensor product of Y is extremely rough. Indeed, this isometric condition characterizes infinite-dimensional spaces in the class of preduals of von Neumann algebras.

Keywords: Banach spaces, slice, homogeneous polynomial, integral polynomial, symmetric projective tensor product, symmetric injective tensor product, C-star-algebra.

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