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On some Curvature-Dependent Steplength for the Gradient Method

The aim of this paper is to show the interest of taking into account the notion of curvature in gradient methods. More precisely, given a Hilbert space H and a strictly convex function $\phi : H \to \mathbb{R}$ of class $\mathcal{C}^2$, we consider the following algorithm

$$(\star) \qquad x_{n+1} = x_n - \lambda_n \nabla\phi(x_n), \qquad \text{with } \lambda_n = \frac{|\nabla\phi(x_n)|^2}{\langle \nabla^2\phi(x_n).\nabla\phi(x_n), \nabla\phi(x_n)\rangle}.$$

We obtain results of linear convergence for the above algorithm, even without strong convexity. Some variants of $(\star)$ are also considered, with different expressions of the curvature-dependent steplength λ_n. A large part of the paper is devoted to the study of an implicit version of $(\star)$, falling into the field of the proximal point iteration. All these algorithms are clearly related to the Barzilai-Borwein method and numerical illustrations at the end of the paper allow to compare these different schemes.

Keywords: Unconstrained convex optimization, steepest descent, gradient method, proximal point algorithm, Barzilai-Borwein stepsize.

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