

**F. Alvarez**

Dep. de Ingeniería Matemática, Centro de Modelamiento Matemático, Universidad de Chile,  
Blanco Encalada 2120, Santiago, Chile  
falvarez@dim.uchile.cl

**J. López**

Dep. de Ingeniería Matemática, Centro de Modelamiento Matemático, Universidad de Chile,  
Blanco Encalada 2120, Santiago, Chile  
jclopez@dim.uchile.cl

**Convergence to the Optimal Value for Barrier Methods Combined  
with Hessian Riemannian Gradient Flows and Generalized Proximal  
Algorithms**

We consider the problem  $\min_{x \in \mathbb{R}^n} \{f(x) \mid Ax = b, x \in \overline{C}, g_j(x) \leq 0, j = 1, \dots, s\}$ , where  $b \in \mathbb{R}^m$ ,  $A \in \mathbb{R}^{m \times n}$  is a full rank matrix,  $\overline{C}$  is the closure of a nonempty, open and convex subset  $C$  of  $\mathbb{R}^n$ , and  $g_j(\cdot)$ ,  $j = 1, \dots, s$ , are nonlinear convex functions. Our strategy consists firstly in to introduce a barrier-type penalty for the constraints  $g_j(x) \leq 0$ , then endowing  $\{x \in \mathbb{R}^n \mid Ax = b, x \in C\}$  with the Riemannian structure induced by the Hessian of an essentially smooth convex function  $h$  such that  $C = \text{int}(\text{dom } h)$ , and finally considering the flow generated by the Riemannian penalty gradient vector field. Under minimal hypotheses, we investigate the well-posedness of the resulting ODE and we prove that the value of the objective function along the trajectories, which are strictly feasible, converges to the optimal value. Moreover, the value convergence is extended to the sequences generated by an implicit discretization scheme which corresponds to the coupling of an inexact generalized proximal point method with parametric barrier schemes. Specializations and simple illustrations of the general results are given for the positive orthant, the unitary simplex and the second-order cone.