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LMI Representations of the Convex Hulls of Quadratic Basic Semialgebraic Sets

We are motivated by the question of when a convex semialgebraic set in $\mathbb{R}^n$ is equal to the feasible set of a linear matrix inequality (LMI). Given a basic semialgebraic set, $\mathcal{V}$, which is defined by quadratic polynomials, we restrict our attention to closure of its convex hull, namely $\overline{\mathbf{co}(\mathcal{V})}$. Our main result is that $\overline{\mathbf{co}(\mathcal{V})}$ is equal to the intersection of a finite number of LMI sets and the halfspaces supporting $\mathcal{V}$ along a particular subset of the boundary of $\mathcal{V}$. As a corollary, we show that in $\mathbb{R}^2$, the halfspaces of concern are finite in number, so that an LMI representation for $\overline{\mathbf{co}(\mathcal{V})}$ always exists.