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A Multiplicity Theorem in $\mathbb{R}^n$

The aim of this paper is to establish the following result:

THEOREM 1. - *Let X be a finite-dimensional real Hilbert space, and let $J : X \to \mathbf{R}$ be a C^1 function such that*

$$\liminf_{\|x\|\to+\infty} \frac{J(x)}{\|x\|^2} \geq 0 \ .$$

Moreover, let $x_0 \in X$ and $r, s \in \mathbf{R}$, with $0 < r < s$, be such that

$$\inf_{x\in X} J(x) < \inf_{\|x-x_0\|\leq s} J(x) \leq J(x_0) \leq \inf_{r\leq\|x-x_0\|\leq s} J(x) \ .$$

Then, there exists $\hat{\lambda} > 0$ such that the equation

$$x + \hat{\lambda} J'(x) = x_0$$

has at least three solutions.

We will proceed as follows. We first give the proof of Theorem 1. Then, we discuss in detail the finite-dimensionality assumption on X. More precisely, we will show not only that it can not be dropped, but also that it is very hard to imagine some additional condition (different from being x_0 a local minimum of J) under which one could adapt the given proof to the infinite-dimensional case. We finally conclude presenting an application of Theorem 1 to a discrete boundary value problem.