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Convex Solids with Planar Homothetic Sections Through Given Points

Extending results of C. A. Rogers ["Sections and projections of convex bodies", Portugal. Math. 24 (1965) 99–103], G. R. Burton ["Sections of convex bodies", J. London Math. Soc. 12 (1976) 331–336] and G. R. Burton and P. Mani ["A characterization of the ellipsoid in terms of concurrent sections, Comment. Math. Helv. 53 (1978) 485–507] to the case of unbounded convex sets, we prove that line-free closed convex sets K_1 and K_2 of dimension n in $\mathbb{R}^n$, $n \geq 4$, are homothetic provided there are points $p_1 \in \operatorname{int} K_1$ and $p_2 \in \operatorname{int} K_2$ such that for every pair of parallel 2-dimensional planes L_1 and L_2 through p_1 and p_2, respectively, the sections $K_1 \cap L_1$ and $K_2 \cap L_2$ are homothetic. Furthermore, if there is a homothety $f : \mathbb{R}^n \to \mathbb{R}^n$ such that $f(K_1) = K_2$ and $f(p_1) \neq p_2$, then K_1 and K_2 are convex cones or their boundaries are convex quadric surfaces. Related results on elliptic and centrally symmetric 2-dimensional bounded sections of convex sets are considered.

Keywords: Homothety, convex body, planar section, quadric surface.

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