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Dynamic Formulation of Optimal Transport Problems

We consider the classical Monge-Kantorovich transport problem with a general cost $c(x,y) = F(y-x)$ where $F\colon \mathbb{R}^d \to \mathbb{R}^+$ is a convex function and our aim is to characterize the dual optimal potential as the solution of a system of partial differential equations.
Such a characterization has been given in the smooth case by L. Evans and W. Gangbo [Mem. Amer. Math. Soc. 653 (1999)] where F is the Euclidian norm and by Y. Brenier [Lecture Notes Math. 1813 (2003) 91–121] in the case where $F = |\cdot|^p$ with $p > 1$. We extend these results to the case of general F and singular transported measures in the spirit of previous work by G. Bouchitté and G. Buttazzo [J. Eur. Math. Soc. 3 (2001) 139–168] using an adaptation of Y. Brenier's dynamic formulation.

Keywords: Wasserstein distance, optimal transport map, measure functionals, duality, tangential gradient, partial differential equations.

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