Journal of Convex Analysis 15 (2008) 439–453

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Pseudometrizable Bornological Convergence is Attouch-Wets Convergence

Let $\mathcal{S}$ be an ideal of subsets of a metric space $\langle X, d\rangle$. A net of subsets $\langle A_\lambda \rangle$ of X is called $\mathcal{S}$-*convergent* to a subset A of X if for each $S \in \mathcal{S}$ and each $\varepsilon > 0$, we have eventually $A \cap S \subseteq A_\lambda^\varepsilon$ and $A_\lambda \cap S \subseteq A^\varepsilon$. We identify necessary and sufficient conditions for this convergence to be admissible and topological on the power set of X. We show that $\mathcal{S}$-convergence is compatible with a pseudometrizable topology if and only if $\mathcal{S}$ has a countable base and each member of $\mathcal{S}$ has an ε-enlargement that is again in $\mathcal{S}$. Further, in the case that the ideal is a bornology, we show that $\mathcal{S}$-convergence when pseudometrizable is Attouch-Wets convergence with respect to an equivalent metric.

Keywords: Bornological convergence, Attouch-Wets convergence, bounded Hausdorff convergence, hyperspace, bornology.

MSC: 54B20; 46A17, 54E35